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Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

If all different, the number of permutations $=n!$; but r things can be permuted in $r!$ ways, and q sets of r things in a set, can be permuted in $(r!)^q$ ways.

$$\therefore s \times (r!)^q = n!, \text{ or } s = \frac{n!}{(r!)^q}.$$

162. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

If $x = \Sigma_0^\infty e^{-k[t+(2a\pi/h)]} \sin n\left(t + \frac{2a\pi}{h}\right)$, find value of x freed from Σ_0^∞.

Solution by the PROPOSER.

Let $2\pi/h = m$. $\therefore x = \Sigma_0^\infty e^{-k(t+am)} \sin n(t+am)$. a can have all positive integral values.

Let $C = \Sigma_0^\infty e^{-akm} \cos(anm)$, $S = \Sigma_0^\infty e^{-akm} \sin(anm)$.

Then $x = e^{-kt}(C \sin nt + S \cos nt)$. Now $C + S\sqrt{(-1)} = \Sigma_0^\infty e^{-am(k+n\sqrt{-1})}$

$$= \frac{1}{1-e^{-m(k+n\sqrt{-1})}} = \frac{1}{1-e^{-km}\cos(mn) - \sqrt{(-1)}^{-km}\sin(mn)}$$

$$= \frac{1-e^{-km}\cos(mn) + \sqrt{(-1)}e^{-km}\sin(mn)}{1-2e^{-km}\cos(mn)+e^{-2km}}.$$

$$\therefore C = \frac{1-e^{-km}\cos(mn)}{1-2e^{-km}\cos(mn)+e^{-2km}}, \quad S = \frac{e^{-km}\sin(mn)}{1-2e^{-km}\cos(mn)+e^{-2km}}.$$

$$\therefore x = \frac{e^{-kt}\{[1-e^{-km}\cos(mn)]\sin nt + e^{-km}\sin(mn)\cos nt\}}{1-2e^{-km}\cos(mn)+e^{-2km}}$$

$$= \frac{e^{-kt} - e^{-k(m+t)}\sin n(t-m)}{1-2e^{-km}\cos(mn)+e^{-2km}}.$$

NOTE ON PROBLEM 145 (UNSOLVED) BY H. S. VANDIVER, STUDENT, UNIVERSITY OF PENNSYLVANIA, PHILADELPHIA, PA.

It is possible to show that

$$F(a, b, c, d) = 2b^2c^2 + 2c^2a^2 + 2a^2b^2 + 2a^2d^2 + 2b^2d^2 + 2c^2d^2 - a^4 - b^4 - c^4 - d^4$$

cannot be expressed as the product of two rational factors. For, assuming that we have

$$F(a, b, c, d) = f(a, b, c, d)f'(a, b, c, d)$$

(by symmetry both f and f' must contain all the letters a, b, c, and d). Put $a = b$, $c = d$. Then

$$F(a,\ a,\ d,\ d) = f(a,\ a,\ d,\ d)f'(a,\ a,\ d,\ d) = 12a^2d^2 - 2a^4 - 2d^4.$$

That is, $12a^2d^2 - 2a^4 - 2d^4$ must be resoluble into two rational factors in a and d, since neither $f(a,\ a,\ d,\ d)$ nor $f'(a,\ a,\ d,\ d)$ can equal unity. It is evident however that $12a^2d^2 - 2a^4 - 2d^4$ does not possess this property.

GEOMETRY.

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190. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Find the locus of the centers of sections of an ellipsoid by planes which are at a constant distance from the center.

Solution by the PROPOSER.

The center of the ellipsoid being the origin, and $(a,\ \beta,\ \gamma)$ being the center of the section, its equation is found to be

$$\frac{a}{a^2}x + \frac{\beta}{b^2}y + \frac{\gamma}{c^2}z - \left(\frac{a^2}{a^2} + \frac{\beta^2}{b^2} + \frac{\gamma^2}{c^2}\right) = 0 \ldots.(1).$$

The perpendicular from the center of the ellipsoid upon it is

$$\left(\frac{a^2}{a^2} + \frac{\beta^2}{b^2} + \frac{\gamma^2}{c^2}\right) \div \sqrt{\left(\frac{a^2}{a^4} + \frac{\beta^2}{b^4} + \frac{\gamma^2}{c^4}\right)} = k,$$

a constant, by the problem. This gives the required locus, which, by rationalizing, is easily seen to be a surface of the fourth degree.

Excellent solutions were also received from *PROFESSORS ZERR*, *WALKER*, and *SCHEFFER*.

CALCULUS.

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151. Proposed by J. SCHEFFER. A. M., Hagerstown, Md.

Integrate the differential equation, $xy\dfrac{\partial^2 z}{\partial x.\partial y} = bx\dfrac{\partial z}{\partial x} + ay.$

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Let $x = e^u,\ y = e^v$; then with $x\dfrac{d}{dx} = \theta,\ y\dfrac{d}{dy} = \theta'$, the given equation reduces to $\theta(\theta' - b)z = ae^v \ldots.(1)$, in which u and v are the independent variables.

The integral of (1) is